Bernoulli's law, kinetic energy and the theory of relativity.

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Bernoulli's law states that if pressure increases along a fluid streamline, then as a consequence, the fluid flow velocity decreases (and vice versa).

The usual formula for Bernoulli's law is:

$$(\rho * v^2)/2 + \rho * g * h + P = const$$

ρ is the density of the liquid;

v is the fluid velocity;

h - height;

P - pressure;

g is the free fall acceleration.

Moreover, the first term in the equation is the kinetic energy of a unit volume of liquid [1].

$$(\rho * v^2)/2$$

And here the question arises: why is there a coefficient 1/2 near the first term?

An explanation is needed here. In the same link there is an elementary derivation of the Bernoulli equation from the law of conservation of energy, and naturally, the kinetic energy formula is used (hence the coefficient).

Kinetic energy, by definition, is the difference between the total energy of the body and the rest energy (according to Einstein's STR) [2].

$$T = (m * c^2) / (1 - v^2/c^2)^{0.5} - (m * c^2)$$

Naturally, the total energy depends on the relativistic mass of the body, which increases with increasing speed.

But, the change in mass can be different, since there is a transverse (v↑, a→) and longitudinal mass (v→, a→) - there is “...the difference in the inertial properties of the body in relation to accelerations in the direction of velocity and perpendicular to it... the inertia of a particle in the direction of velocity (longitudinal mass) is greater than perpendicular to the velocity (transverse mass)...” [3].

Transverse mass:

$$m (v\uparrow, a\rightarrow) = m / (1 – v^2/c^2)^{0.5}$$

Longitudinal mass:

$$m (v\rightarrow, a\rightarrow) = m / (1 – v^2/c^2)^{1.5}$$

Since in Bernoulli's law the flow velocity and force/pressure coincide in direction, the kinetic energy will be equal to ($v\rightarrow$, $a\rightarrow$):

$$T = (3 * m * v^2)/2$$

$$(T = (m * c^2) / (1 - v^2/c^2)^{1.5} - (m * c^2))$$

So, in the Bernoulli equation, the coefficient 3/2 should be near the first term.

$$(3 * \rho * v^2)/2$$

Explanation. Kinetic energy can be expanded into a Maclaurin series (total energy minus rest energy), so depending on the mass (transverse, longitudinal) there will be different coefficients: 1/2 or 3/2.

Let's demonstrate the Maclaurin expansion for these two cases, since the classical formula for kinetic energy is simply the first term of the Maclaurin expansion (when ($v\uparrow$, $a\rightarrow$)).

$$T = (m * v^2)/2$$

In our case, we have two similar formulas:

1. $(1 – v^2/c^2)^{(-1/2)}$

(From the formula $m (v\uparrow, a\rightarrow) = m / (1 – v^2/c^2)^{0.5} = m * (1 – v^2/c^2)^{(-1/2)}$)

2. $(1 – v^2/c^2)^{(-3/2)}$

(From the formula $m (v\rightarrow, a\rightarrow) = m / (1 – v^2/c^2)^{(3/2)} = m * (1 – v^2/c^2)^{(-3/ 2)}$)

Therefore, when expanded in a series according to Maclaurin, we get.

1st case ($v\uparrow$, $a\rightarrow$).

$$(1 – v^2/c^2)^{(-1/2)} = 1 + v^2/(2 * c^2) + (3 * v^4)/(8 * c^4) + (5 * v^6)/(16 * c^6) + \dots$$

Then, the expression for the kinetic energy in the 1st case is as follows.

$$T = (m * c^2) * (1 – v^2/c^2)^{(-1/2)} - (m * c^2) = (m * c^2) * ((1 – v^2/c^2)^{(-1/2)} - 1)$$

$$T = (m * v^2)/2 + (3 * m * v^4)/(8 * c^2) + (5 * m * v^6)/(16 * c^4) + \ldots$$

If we take into account only the first term of the expansion (speed much less than the speed of light), then we obtain the classical formula for the kinetic energy:

$$T \approx (m * v^2)/2$$

2nd case (v→, a→).

$$(1 – v^2/c^2)^{(-3/2)} = 1 + (3 * v^2)/(2 * c^2) + (15 * v^4)/(8 * c^4) + (35 * v^6)/(16 * c^6) + ...$$

Therefore, for the 2nd case, we obtain a formula for the kinetic energy with other coefficients.

$$T = (m * c^2) * (1 – v^2/c^2)^{(-3/2)} - (m * c^2) = (m * c^2) * ((1 – v^2/c^2)^{(-3/2)} - 1)$$

$$T = (3 * m * v^2)/2 + (15 * m * v^4)/(8 * c^2) + (35 * m * v^6)/(16 * c^4) + \ldots$$

We take into account only the first term of the expansion (speed much less than the speed of light):

$$T \approx (3 * m * v^2)/2$$

If for the kinetic energy we take into account the first term of the expansion into a series, then the first term of the Bernoulli equation should have a coefficient of 3/2.

$$(3 * \rho * v^2)/2$$

Taking into account the new coefficient, Bernoulli's law will be written in the form:

$$(3 * \rho * v^2)/2 + \rho * g * h + P = const$$

In the end, I note that it is Bernoulli's law that explains why the fluid velocity is higher in the narrow part of the pipe, and the pressure is less than in the pipe section of a larger diameter [4].

1. Bernoulli's principle. Wikipedia (ru). https://en.wikipedia.org/wiki/Bernoulli%27s_principle
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4. Venturi effect. Wikipedia. https://en.wikipedia.org/wiki/Venturi_effect